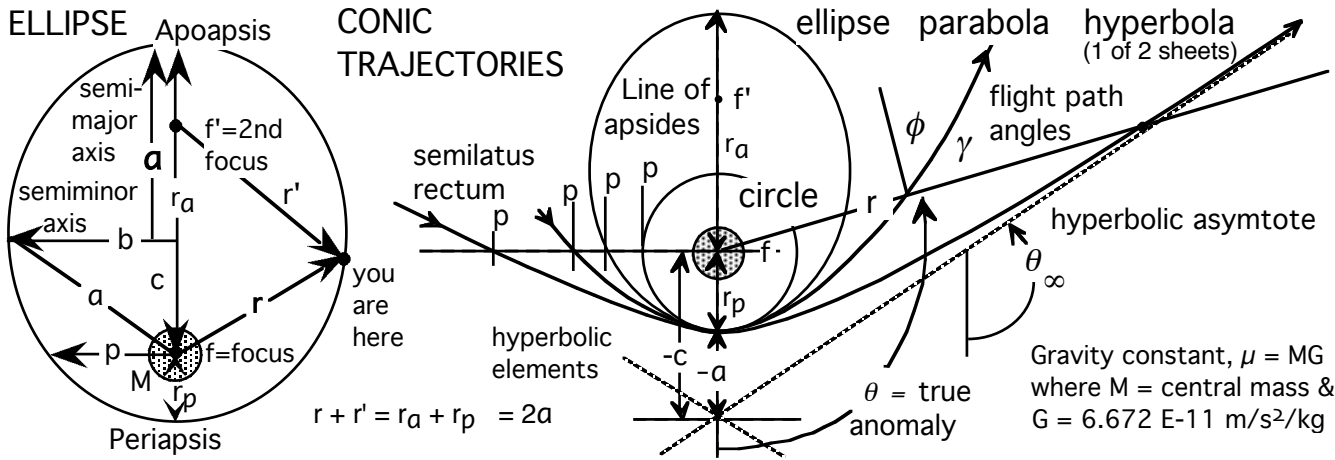


# Basic Keplerian Orbital Dynamics - Review Sheet

by Gerald David Nordley

gnordley@gdnordley.com rev.5 ©2011



**For all Conics:**  $r = r_p(1+e)/(1+e \cos \theta) = p/(1+e \cos \theta)$

$r_p = p/(1+e) = -\mu(1-e)/2E \quad \cos \theta = (p/r - 1)/e$

$h = r v \cos \phi = r v \sin \gamma = r_a v_a = r_p v_p = (p \mu)^{1/2}$

$p = h^2/\mu = (r_p v_p)^2/\mu = r_p(1+e) = r(1+e \cos \theta)$

$E = v^2/2 - \mu/r = -\mu/2a = -\mu(1-e)/2r_p \quad a = -\mu/2E$

$e = (1 + 2Eh^2/\mu^2)^{1/2} = (p/r_p) - 1 = (1 - p/a)^{1/2} = (1 - (b/a)^2)^{1/2}$

$v^2 = 2(E + \mu/r) = (2/r - 1/a)\mu \quad v_p^2 = (1+e)\mu/r_p$

**For Circle:**  $v_c^2 = \mu/r$       **For Parabola:**  $v_e^2 = 2\mu/r$

**For Ellipse:**  $r_a = a(1+e) \quad r_p = a(1-e) \quad a^2 = b^2 + c^2$

$v_a^2 = (1-e)\mu/r_a \quad v_a/v_p = r_p/r_a \quad p = a(1-e^2)$

$e = (r_a - r_p)/2a = (v_p - v_a)/(v_p + v_a) = 1 - v_a^2/(\mu/r_a)$

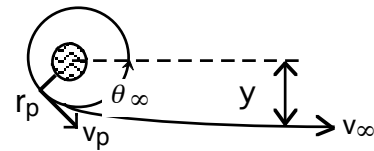
$r_p = r_a(1-e)/(1+e) \quad E = -\mu(1+e)/2r_a \quad a = -\mu/2E = 1/(2/r - v^2/\mu)$

$T = 2\pi(a^3/\mu)^{1/2} \quad T_1/T_2 = (a_1/a_2)^{3/2}(\mu_1/\mu_2)^{1/2} \quad n = (\mu/a^3)^{1/2}$

$\tan \theta = \tan \phi / (1 - r/p) \quad \tan \phi = e \sin \theta / (1 + e \cos \theta)$

**For Hyperbola:**  $v_\infty = (v^2 - 2\mu/r)^{1/2} = (v^2 - v_e^2)^{1/2}$

$\cos \theta_\infty = -1/e \quad y = (a - r_p) \sin(\pi - \theta_\infty) \quad C_3 = v^2 - v_e^2$



**Time of Flight Equations:**  $t_\theta =$  Time to reach  $\theta$  from periapsis,      **For Circle:**  $t_\theta = (T/2\pi)\theta$

**Ellipse:**  $\tan E = (1 - e^2)^{1/2} \sin \theta / (e + \cos \theta) \quad \cos \theta = (e - \cos E) / (e \cos E - 1)$

$t_\theta = (a^3/\mu)^{1/2} (E - e \sin E) = (T/2\pi)(E - e \sin E) \quad t_\pi = T/2 = \pi/n = \pi(a^3/\mu)^{1/2}$

$M = 2\pi t_\theta/T = (E - e \sin E)$  [To find  $E$  from  $M$ , iterate  $E_i = M + e \sin E_{i-1}$  until  $E_i \approx E_{i-1}$ .]

**Parabola:**  $D = p^{1/2} \tan(\theta/2) \quad t_\theta = (1/\mu)^{1/2} (pD + D^3/3)/2 \quad D =$  parabolic eccentric anomaly

$t_\theta = ((2r_p)^3/\mu)^{1/2} (U + U^3/3)$  where  $U = \tan(\theta/2)$

**Hyperbola:**  $\cosh F = \cos E = (e + \cos \theta) / (1 + e \cos \theta) \quad F =$  hyperbolic eccentric anomaly

$t_\theta = (-a^3/\mu)^{1/2} (e \sinh F - F) \quad \text{asymptote: } \cos \theta_\infty = -1/e \quad \cosh F = (1 - r/a)/e$

note:  $\sinh(A) = [e^A - e^{-A}]/2$ ;  $\cosh(A) = [e^A + e^{-A}]/2$